Achieving Minimum Coverage Breach under Bandwidth Constraints in Wireless Sensor Networks

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Presented by L. C. Chen

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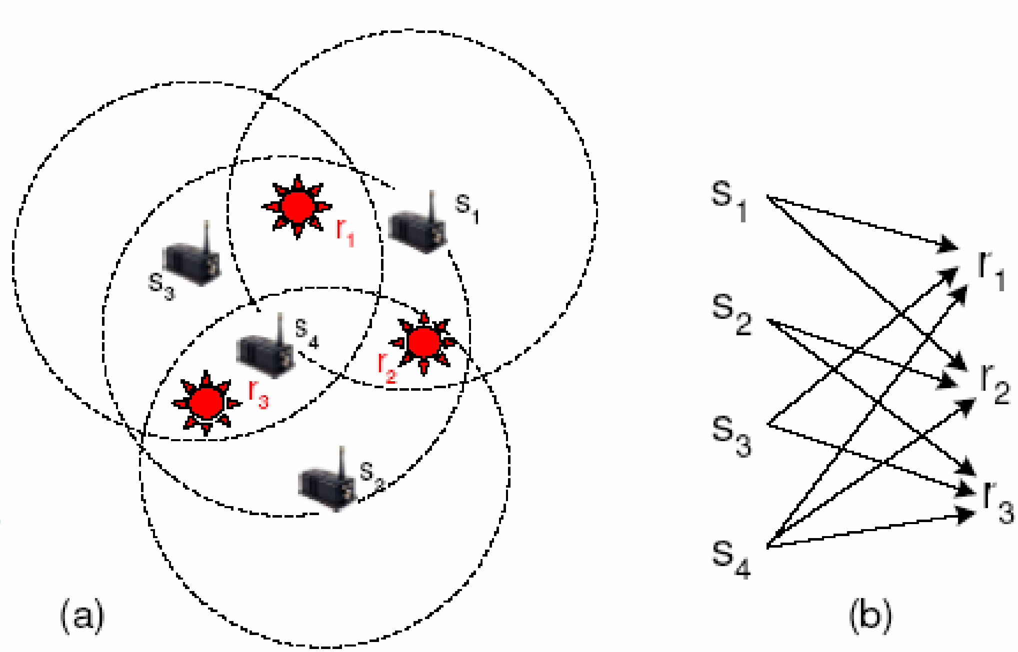
Outline

- Introduction
- Minimum Breach Problem in Sensor Networks
  - Problem Definition
  - Complexity Classification of the Minimum Breach Problem
- Approximation Algorithms
  - Integer Programming Formulation of the Minimum Breach Problem
  - Heuristic I: RELAXATION
  - Heuristic II: MINBREACH
- Simulation Study
- Conclusion and Extensions

Introduction

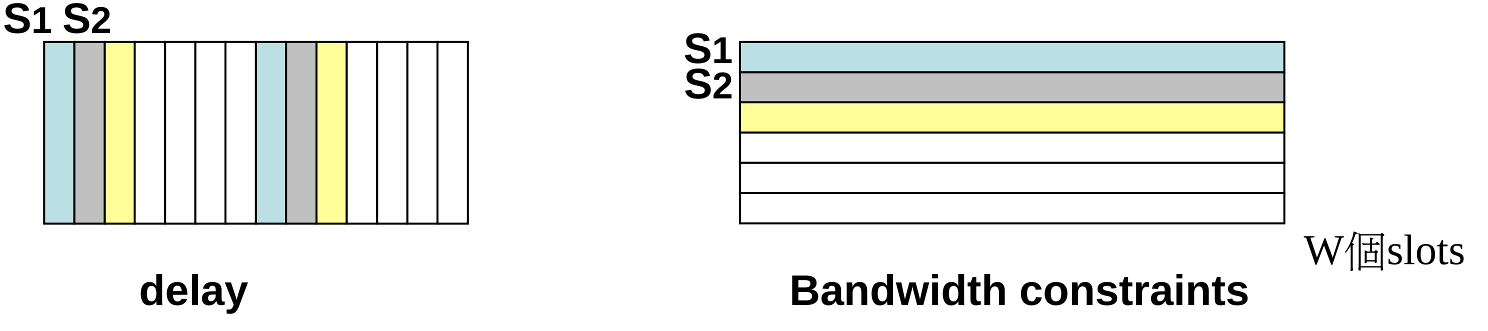
- Stochastically deployed sensor network.
- Oscillated between active modes and inactive modes.
- Divided into mutually exclusive subsets without considerate on subset sizes.

{S1,S4}
{S2,S3}



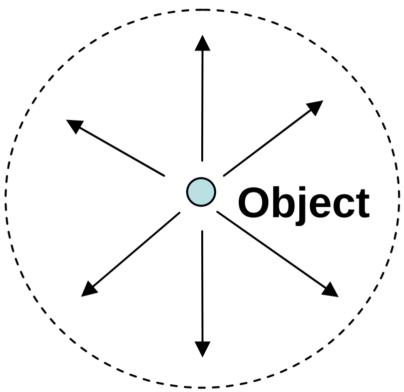
Why are Bandwidth Constraints?

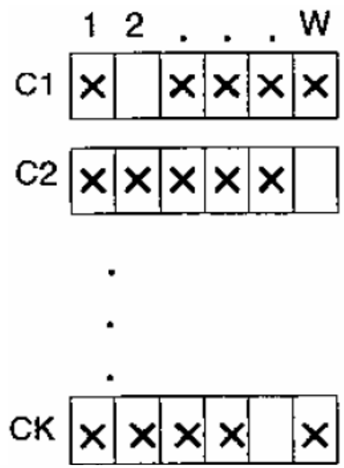
- Each active sensor will send the sensory data directly to the base station.
- “Bandwidth” is the total number of time slots.
- The total number of sensors simultaneously sending to the base station must be restricted by the bandwidth.



Problem Definition

- Object has equal chance of being detected from all direction.
- If sensors lie within the area boundary, the object is considered covered.



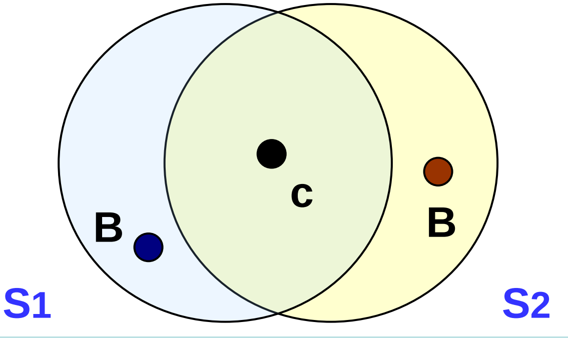


Sensor organization to satisfy the bandwidth constraints

- PROBLEM: MINIMYM BREACH
- INSTANCE: A collection S of sensors, a collection A of targets, and the sensor-target coverage map.
- QUESTION: Can we divide S into disjoint subsets such that the overall breach is at most B and each subset has at most W sensors in it?

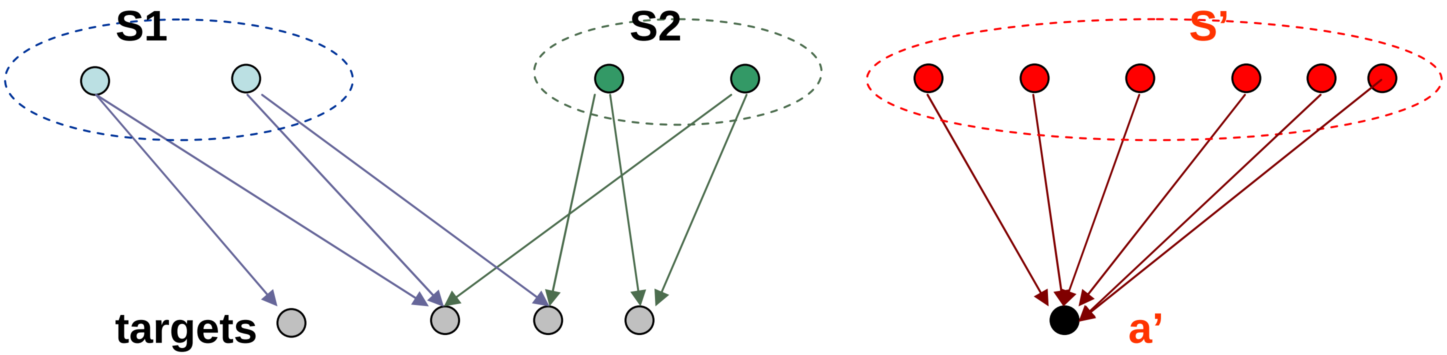
Complexity Classification of the Minimum Breach Problem —*MINIMUM BREACH*

- Prove that MINIMUM BREACH problem is NP-complete.
- Divide the sensors into two disjoint subsets to minimize the overall breach.—*MINIMUM 2SET BREACH problem*
- If the total breach is at most B, the corresponding solution {S1}U{S2} guarantees that the cardinality of the subsets in C that are split is at least |C|-B.



MINIMUM 2-W BREACH problem

- Make W=|S|+1, and |S'|=2W-|S|.
- MINIMUM 2SET BREACH can be satisfied if and only if MINIMUM 2-W BREACH can be satisfied.
- MINIMUM 2-W BREACH is a class of MINIMUM BREACH where the number of subset is restricted to 2.



Integer Programming Formulation of the Minimum Breach Problem

i	the i^{th} sensor, when used as a subscript;
j	the j^{th} target, when used as a subscript;
k	the k^{th} subset, when used as a subscript;
$x_{k,i}$	variable, $x_{k,i} = 1$ if the k^{th} subset includes sensor i, otherwise $x_{k,i} = 0$;
$y_{k,j}$	variable, $y_{k,j} = 1$ if the k^{th} subset covers target j, otherwise $y_{k,j} = 0$;
K	the upper bound for the total number of subsets;
W	bandwidth, used as the upper bound for subset sizes;
N	the number of sensors;
M	the number of targets;
$a_{i,j}$	$a_{i,j} = 1$ if sensor i covers target j, otherwise $a_{i,j} = 0$.

Integer Programming Formulation of the Minimum Breach Problem

$$\min\{\sum_{k=1}^{K}\sum_{j=1}^{M}(1-y_{k,j})\} \quad (1)$$

To minimize the un-covered objects
→ to maximize the covered objects

$$\sum_{i=1}^{N} a_{i,j}x_{k,i} \geq y_{k,j}, \quad \forall j = 1..M,\ k = 1..K; \quad (2)$$

Object j is covered by sensor i and sensor i is belonged to set k
→ object j is covered by set k

$$\sum_{k=1}^{K} x_{k,i} = 1, \quad \forall i = 1..N; \quad (3)$$

Each set is mutually exclusive.

$$\sum_{i=1}^{N} x_{k,i} = W, \quad \forall k = 1..K; \quad (4)$$

Each set can't contain the number of sensor more then W.

$$y_{k,j} \in \{0,1\}, \quad \forall k = 1..K,\ j = 1..M; \quad (5)$$

$$x_{k,i} \in \{0,1\}, \quad \forall k = 1..K,\ i = 1..N. \quad (6)$$

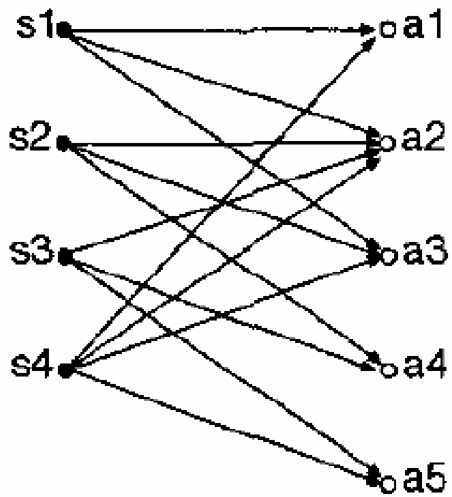
Heuristic I: RELAXATION

- First: the Integer Programming problem is relaxed to a Linear Programming problem, and an optimal solution for LP is computed.

$$0 \le y_{k,j} \le 1, \qquad \forall k = 1..K, \quad j = 1..M; \qquad (5')$$

$$0 \le x_{k,i} \le 1, \qquad \forall k = 1..K, \quad i = 1..N. \qquad (6')$$

- Second: a greedy algorithm is employed to find an integer solution based on the optimal solution obtained at the first set.
- Third: the solution from problem is used to construct the subsets.



(a) bipartite graph

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \end{pmatrix}$$

(b) coverage matrix $(a_{i,j})$

Fig. 2. An example: 4 sensors, 5 targets, with bandwidth=2

$$y_{1,j} = 1, j = 1..5, \quad \{x_{1,1}\ x_{1,2}\ x_{1,3}\ x_{1,4}\} = \{0\ \ 1\ \ 0\ \ 1\}$$

$$y_{2,j} = 1, j = 1..5, \quad \{x_{2,1}\ x_{2,2}\ x_{2,3}\ x_{2,4}\} = \{1\ \ 0\ \ 1\ \ 0\}$$

So we get optimal solution $C1 = \{s_2, s_4\}$ and $C2 = \{s_1, s_3\}$.

Heuristic II: MINBREACH

- I_1: denote the rows in the upper part {0,1,-1}
- I_2: denote the rows in the lower part {0,1}
- J_X: represent the columns that correspond to the $\{x_{k,i}\}$ in the original (IP)
- J_Y: represent the columns that correspond to the $\{y_{k,j}\}$ in the original (IP)

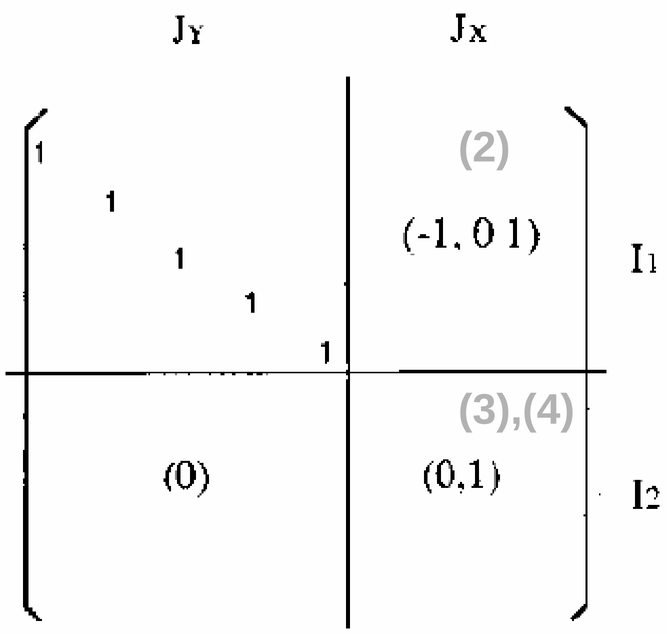
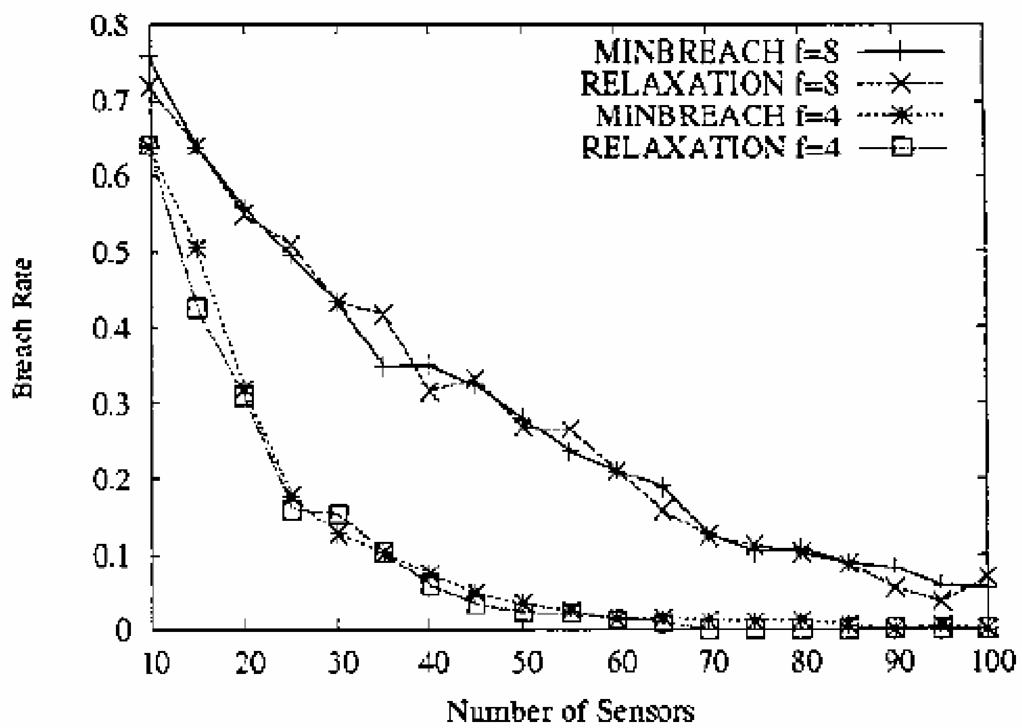


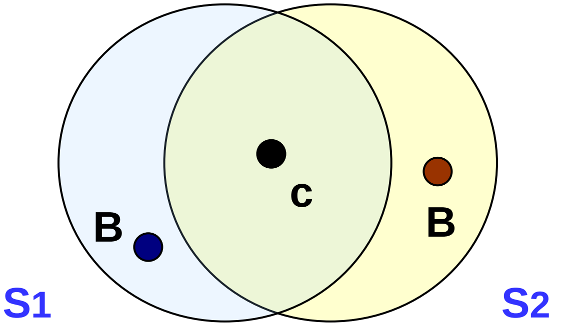
Fig. 3. Coefficient matrix A in $Ax \leq b$

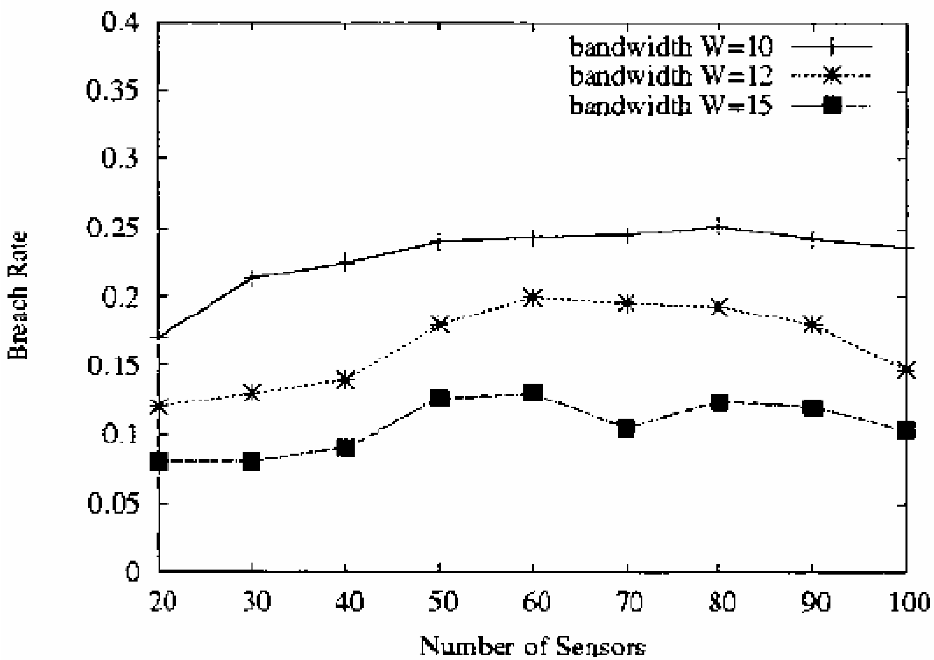
Simulation Study

- As the number of targets increase from 10 to 100, the number of sensors also increase from 10 to 100, and bandwidth increase from 2 to 20.
- It verifies that higher f leads to higher breach rate.

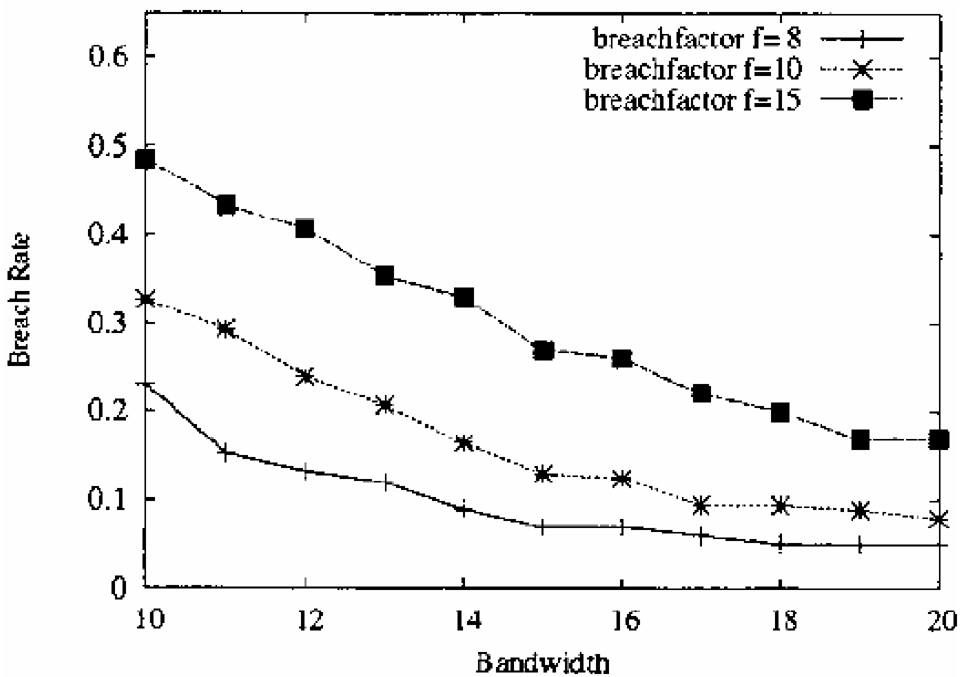


Comparison of RELAXATION and MINBREACH





(a) Bandwidth constraint is the limiting factor when more sensors join the network; increasing bandwidth can significantly decrease the breach rate



(b) Effect of increasing bandwidth

Conclusion and Extensions

- To improve sensor coverage, deploying more sensors must be accompanied by increasing bandwidth, otherwise, the coverage may be decreased as a result.
- To minimize the maximal breach is also an NP-complete problem that requires efficient approximation algorithm.

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