Bounds on the Benefit of Network Coding: Throughput and Energy Saving in Wireless Network

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Introduction

- In recent years, network coding has become an important research topic in network information theory.
  - Provide guidelines for designing or improving efficient and high performance wired or wireless networks

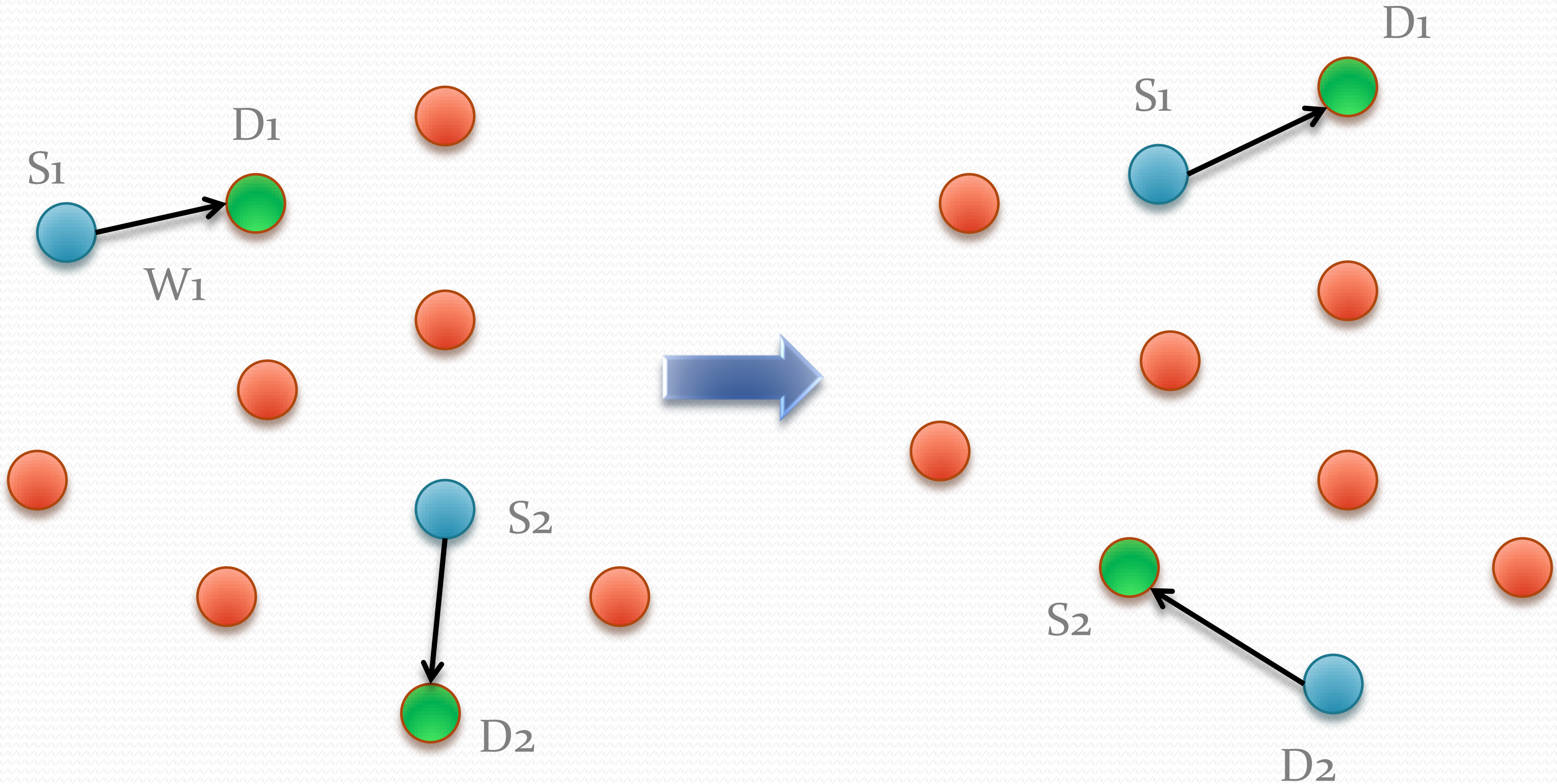
Introduction

- This paper: Study the fundamental limitation of network coding.
  - Energy gain of network coding for a single multicast session.
  - Throughput gain of network coding for single multicast session.
  - Energy gain for multiple unicast session.
  - Throughput gain for multiple unicast session.

Model

- A communication network is a collection of directed (or undirected) links connecting communication nodes.
- The links established through wireless transmissions.
- Assume that the topology has connectivity: needed for establishing the demanded session
- Assume that the nodes are distributed in d-dimensional Euclidean space
- denote the set of node by V

Model – Wireless Channel



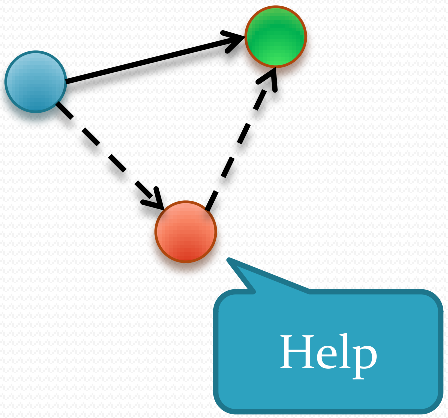
SD := {(S1, D1), (S2, D2), ….., (Sm, Dm)}

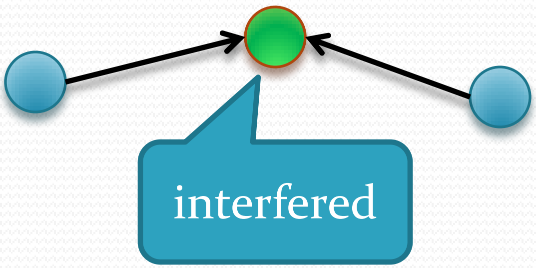
Model – Wireless Channel

- | Si – Di | is the *Euclidean distance* between the nodes Si and Di.
- S := { S1, S2, …., Sm } is the set of transmitter.
- The assigned transmission rate from node Si to node Di is Wi = W for successful transmission where W is channel capacity.

Model – Wireless Channel

- Broadcast channel
  - Broadcasting data to all neighbors can help to increase the throughput.
  - Simultaneous reception from different nodes is not feasible



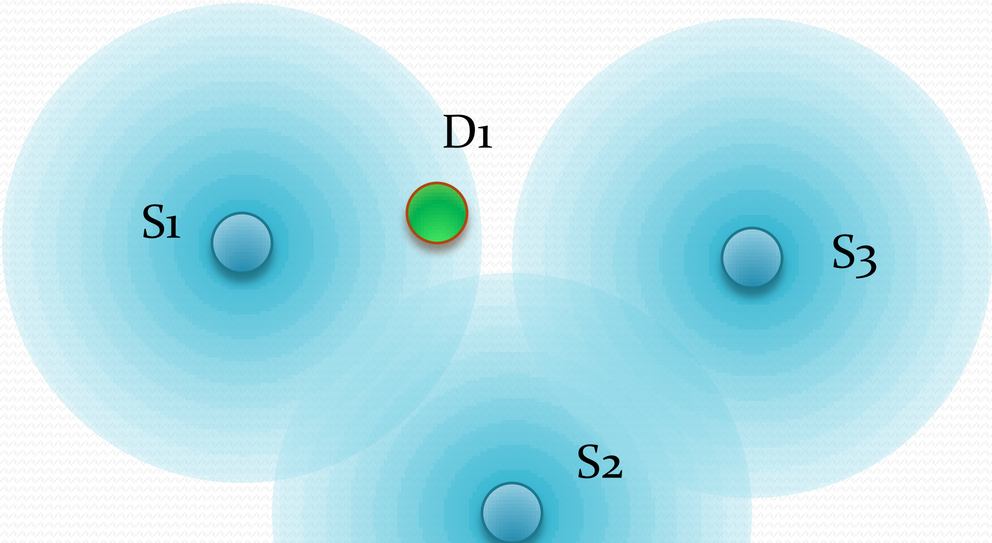


Model – Wireless Channel

- Protocol model
  - Transmission is modeled as successful if

$$|S_k - D_i| \geq (1+\Delta) r, \forall S_k \in S - \{S_i\}$$

$$|S_i - D_i| \leq r$$



- Physical Model

$$\mathrm{SINR} = \frac{PG_{ii}}{N_o + \sum_{k \neq i, k \in \mathcal{S}} PG_{ki}} \geq \beta$$

$$G_{ki} = |S_k - D_i|^{-\alpha}$$

Model – Connectivity Graph and Traffic Pattern

- Wireless networks: modeled by geometric graphs.
- Source-Terminal pair:

$$\mathcal{AB} := \{(A_1, B_1), ..., (A_k, B_k)\}$$

- Hop-count between two nodes

$$\ell(A_i, B_j)$$

$$\ell(\mathcal{A}, B_j) = \min\{\ell(A_i, \breve{B}_j) : A_i \in \mathcal{A}\}$$

Model – Transport Capacity

- Unit of transport capacity is “bit-meter per second”

$$C_T(\mathcal{A}\mathcal{B}) := \max_{\text{multi-hop paths}} \sum_k |A_k - B_k| R_k$$

Model – Data Stream and Energy Consumption

- Non-coding
  - Flow scheme: replication, forwarding
- Coding Scheme
  - Flow scheme + allowing the packets to be decoded or recoded at each node where they received
- Consumed energy for transporting information is proportional to the number of transmissions.

Model – Data Stream and Energy Consumption

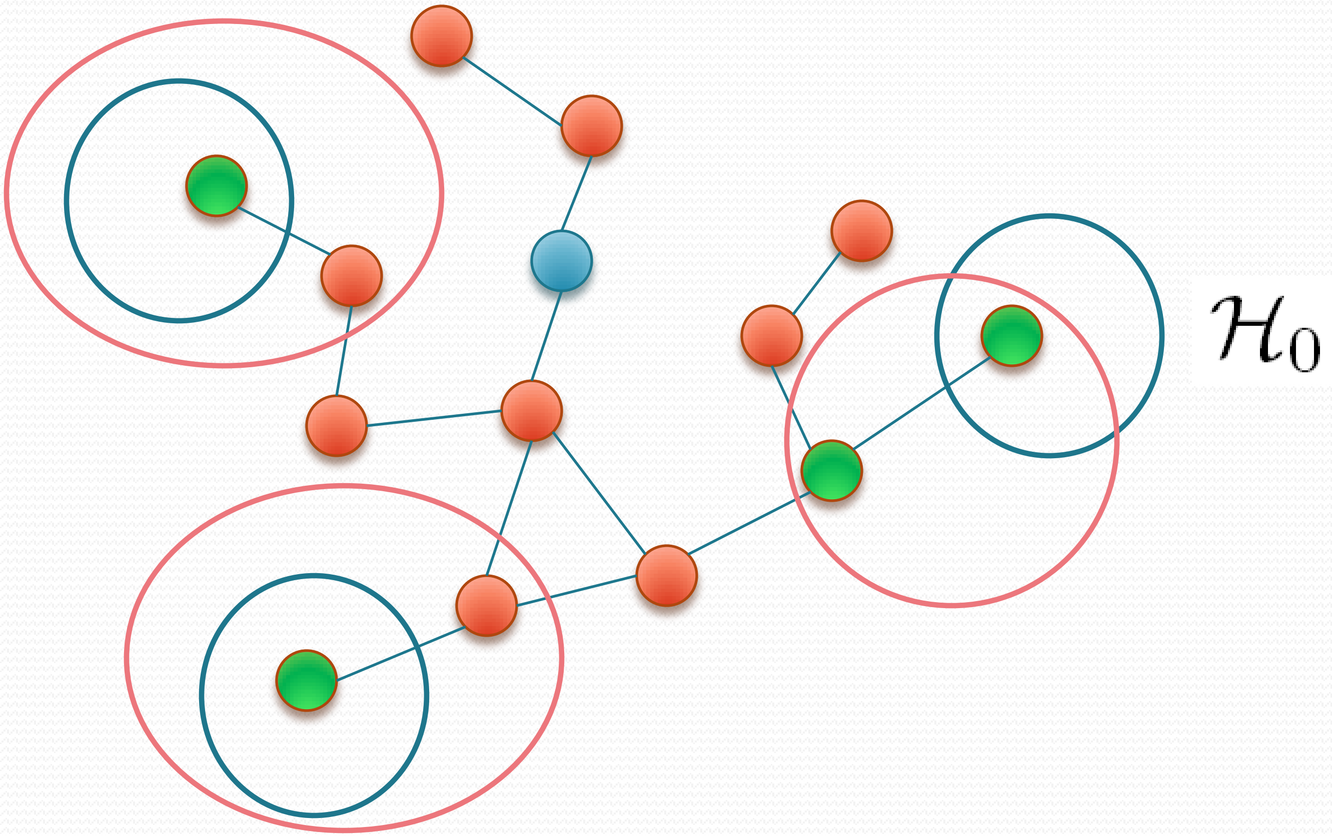
- Summarize the assumptions

  A1. (Optimality source-coded data): Each source generates a data stream in an optimal compressed format.

  A2. (Independence of information for different sources): Different sources generates independent information.

  A3. (Energy consumption): The consumed energy for transporting the information is proportional to the number of transmissions and each transmission delivers 1 bit.

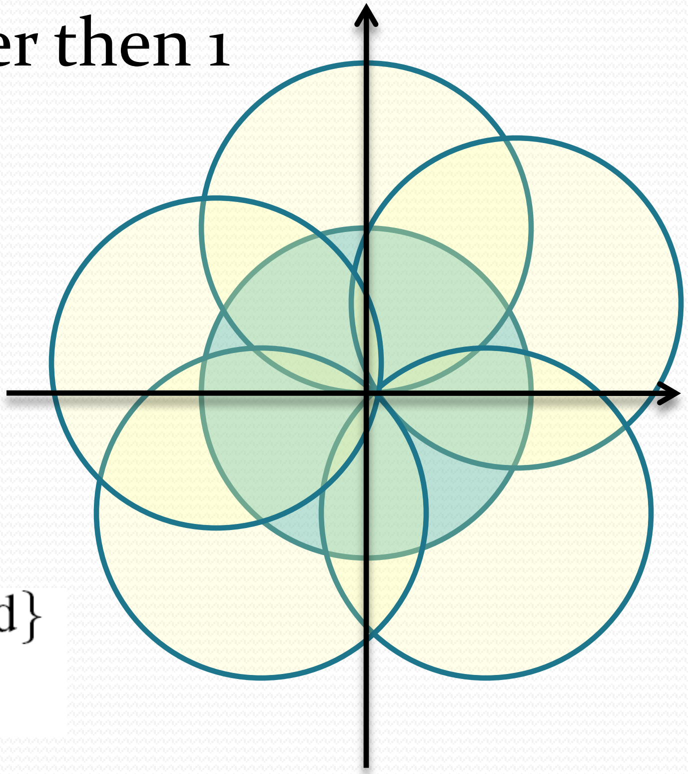
Single Multicast



Single Multicast

- The maximum number of nodes that can be placed in the unit d-dimemsional sphere, such that every two of them have a distance strictly larger then 1

$$\kappa_d := \begin{cases} 2 & \text{if } d = 1 \\ 5 & \text{if } d = 2 \\ 13 & \text{if } d = 3 \end{cases}$$



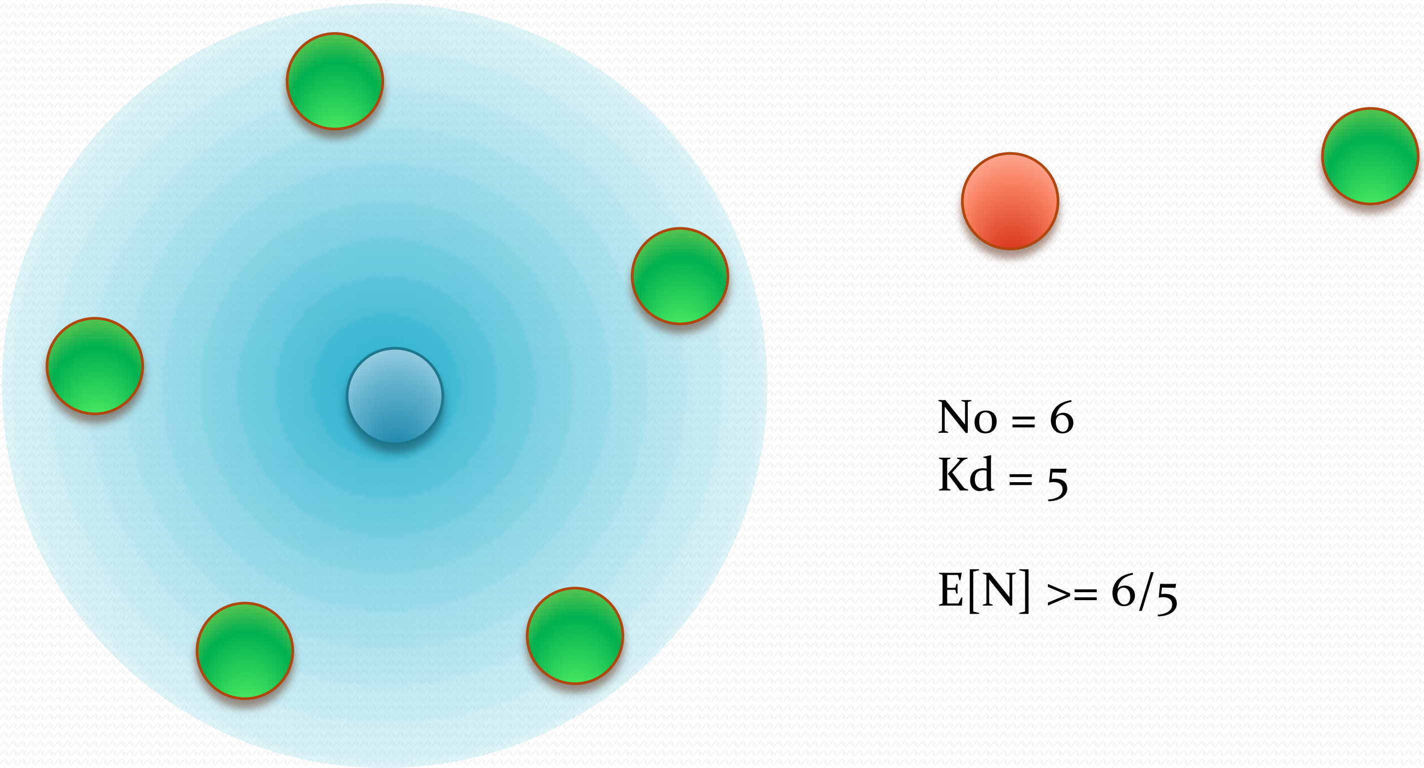
$$k := \min\{j : \text{the nodes of } \mathcal{H}_j \text{ are connected}\}$$
$$n_i := \#\{\text{connectivity components of } \mathcal{H}_i\}$$

Single Multicast – Lemma 1

- Lemma 1: Denote the number of transmissions for sending a multicast bit under an arbitrary coding scheme by N. Then under assumptions A1 and A3,

$$\mathbb{E}[N] \geq n_0/\kappa_d$$

Single Multicast – Lemma 1



No = 6
Kd = 5

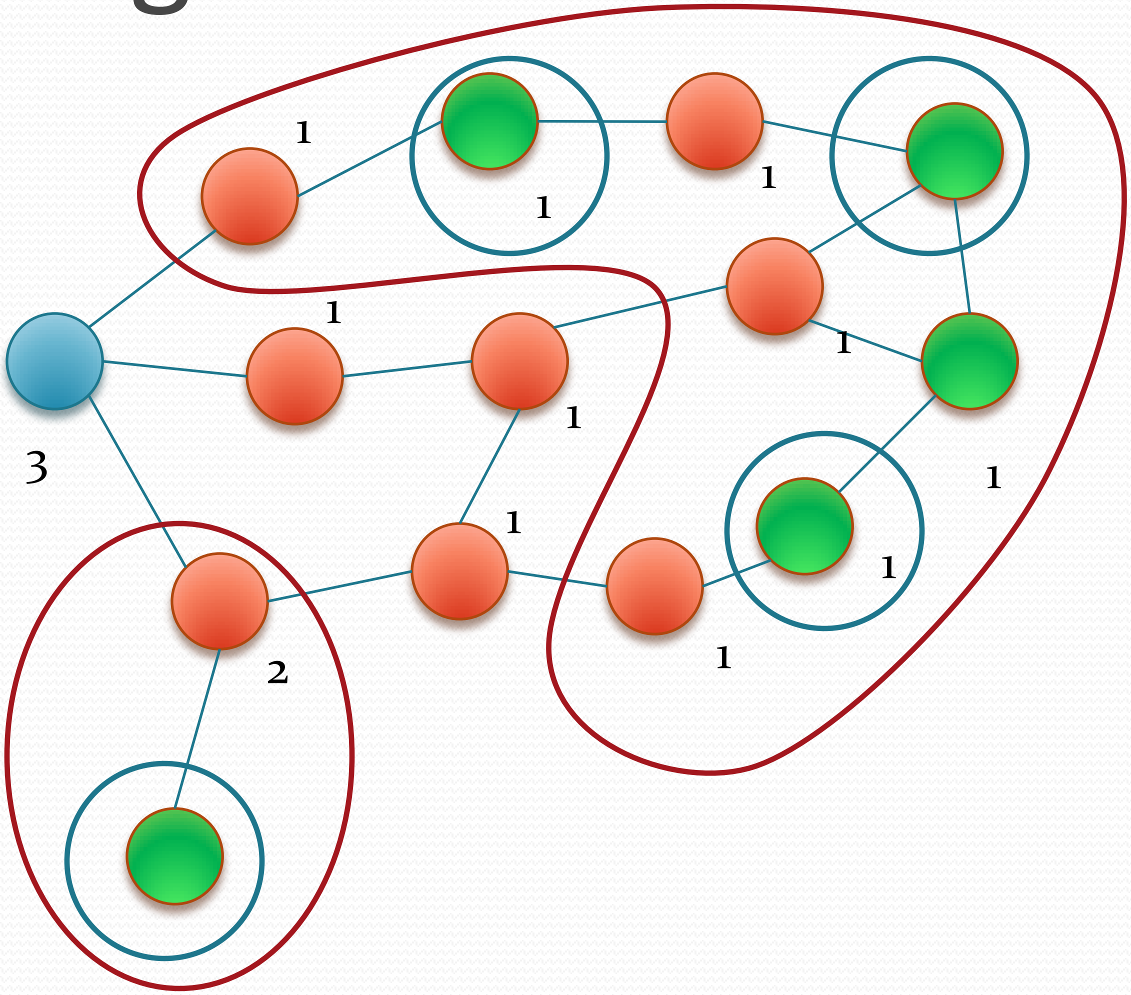
E[N] >= 6/5

Single Multicast – Lemma 2

- Denote the number of transmissions for sending a multicast bit under an arbitrary coding scheme by N. Then under assumptions A1 and A3

$$\mathbb{E}[N] \geq n_1 + \ldots + n_k$$

Single Multicast – Lemma 2



$$n_0 = 4$$

$$n_1 = 2$$

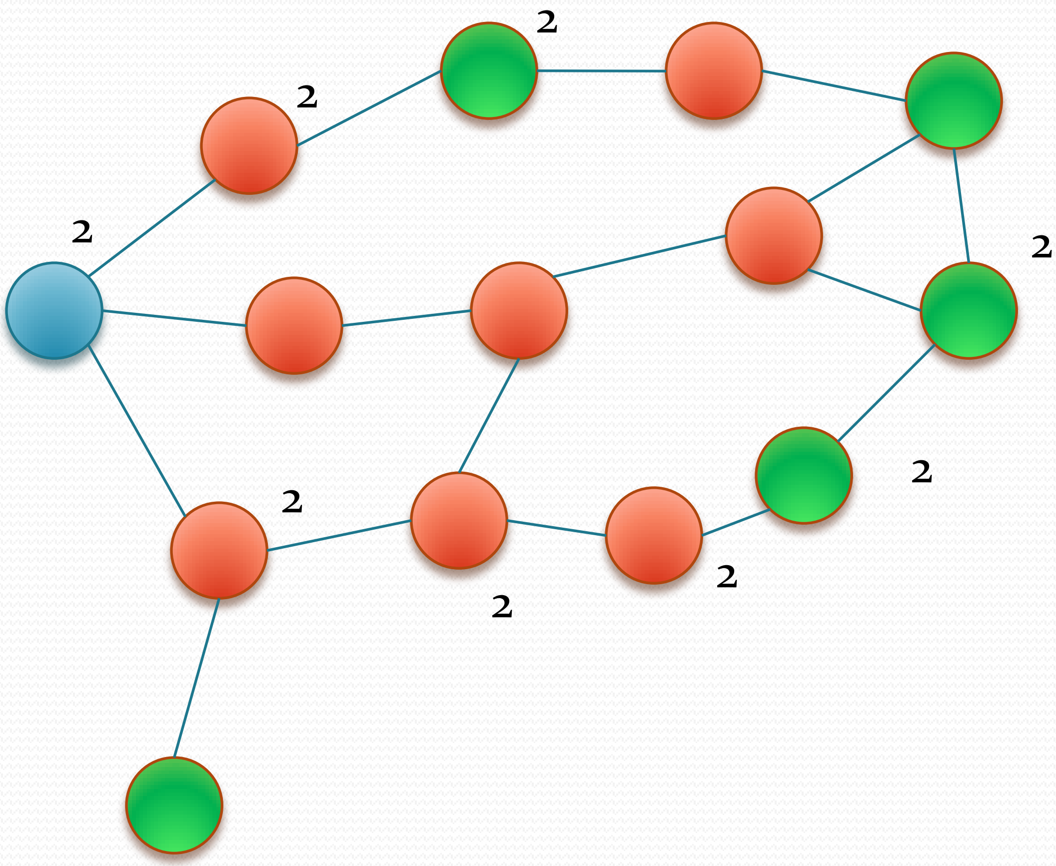
$$n_2 = 1$$

Single Multicast – Lemma 3

- There exists a flow scheme for the multicast session which transports every bit using N' transmissions, where

$$N' \leq 3n_0 + 2(n_1 + \ldots + n_k) - 2k - 1$$

Single Multicast – Lemma 3



$n_0 = 4$

$n_1 = 2$

$n_2 = 1$

Single Multicast – Theorem 1

- Consider a single multicast session which is optimally source coded in an arbitrary wireless network. The gain of network coding in terms of reducing the expected number of transmissions is less than a factor of $3\kappa_d+2$.

$$\frac{N'}{\mathrm{E}[N]} < 3\kappa_d + 2$$

Single Multicast – Throughput Gain

- Using the result of [16], [17], applied the physical model and SINR model

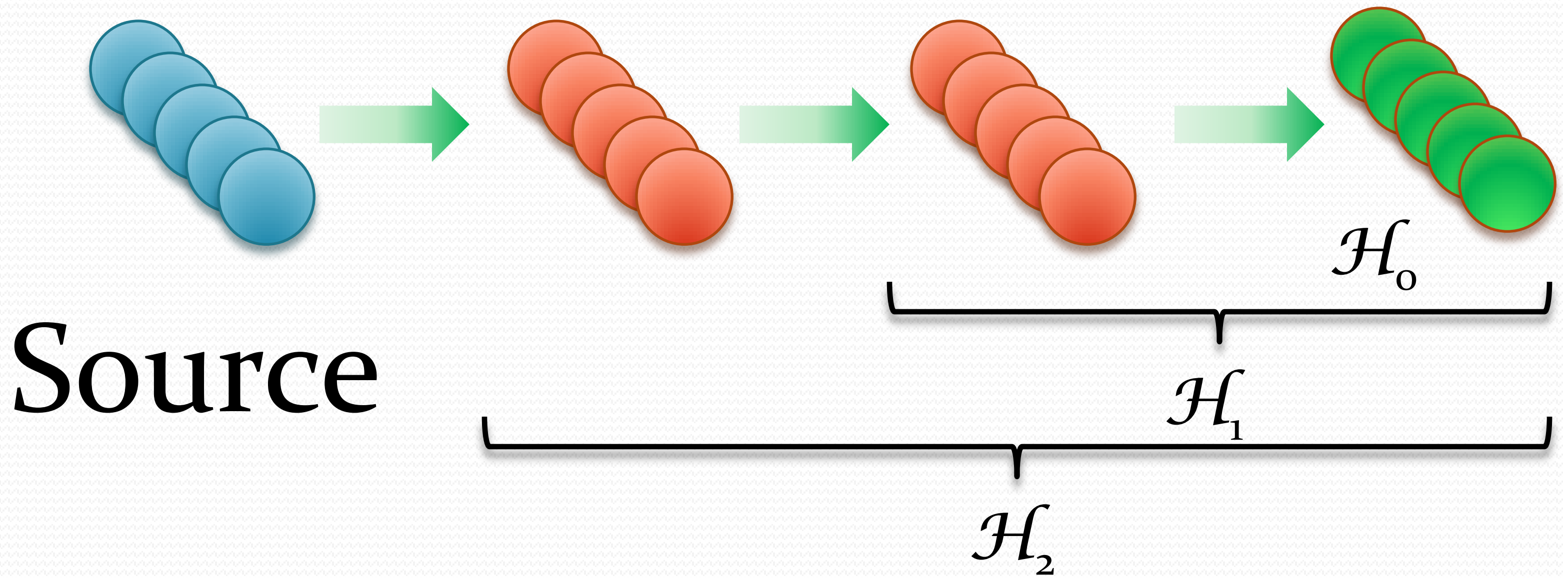
$$c := \begin{cases} \lceil 2 + (1+\Delta)\sqrt{d} + 3 \rceil^{d} \\ (5^{d} - 2^{d}) \left\lceil \sqrt{d} \left(2 + \left(\frac{\beta \sum_{J \in \mathbb{Z}^{d}}^{|J|>0} |J|^{-\alpha}}{1-\rho} \right)^{\frac{1}{\alpha}} \right) \right\rceil^{d} \end{cases}$$

Multiple Unicast – Theorem 3

- Assume that $b_1, b_2, \ldots, b_k$ are optimally source coded bits of some simultaneous unicast sessions. Denote N as the number of transmissions used by an arbitrary coding scheme for transporting these bits. Also, denote the source and terminal of bit b_j by A_j and B_j for $j = 1, \ldots, k$ (a node can be the source or destination for several bits). Then,

$$\mathbb{E}[N] \geq \max\Big[\sum_{j=1}^{k} \ell(A_j, \mathcal{B}), \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)\Big]$$

Multiple Unicast – Theorem 3



Denote the number of sources which are not in $\mathcal{H}_i$ by q_i.

$$q_i = \sum_j \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]}$$

Multiple Unicast – Theorem 3

$$\mathbb{E}[N] \geq \sum_{i=0}^{\infty} q_i = \sum_{i=0}^{\infty} \sum_{j=1}^{k} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]}$$

$$= \sum_{j=1}^{k} \sum_{i=0}^{\infty} \mathbb{I}_{[\ell(A_j,\mathcal{B})>i]} = \sum_{j=1}^{k} \ell(A_j,\mathcal{B})$$

Multiple Unicast – Theorem 3

- Assume that $b_1, b_2, \ldots, b_k$ are optimally source coded bits of some simultaneous unicast sessions. Denote N as the number of transmissions used by an arbitrary coding scheme for transporting these bits. Also, denote the source and terminal of bit b_j by A_j and B_j for $j = 1, \ldots, k$ (a node can be the source or destination for several bits). Then,

$$\mathbb{E}[N] \geq \max\left[\sum_{j=1}^{k} \ell(A_j, \mathcal{B}), \sum_{j=1}^{k} \ell(\mathcal{A}, B_j)\right]$$

Multiple Unicast – Corollary 2

- Consider multiple unicast sessions with optimally source coded data stream. For the following two scenarios, there is no network coding benefit in terms of the number of transmissions (energy).
  1. A set of sources send their independent information to a single sink.
  2. A single source sends independent information to eachof a set of terminals.

Multiple Unicast – Theorem 4

- Denote the maximum transport capacity of an arbitrary wireless network by using the flow scheme and coding scheme by C_T^f and C_T^{nc} respectively. Then,

$$C_T^{\mathrm{nc}} \leq K_d \cdot C_T^{\mathrm{f}}$$

where $K_d = 2$ if $d = 1$ and $K_d = \pi$ if $d = 2, 3$ (d is the dimension of the space).

Multiple Unicast – Theorem 4

- Proof:

By Lemma 4, there exists an unit vector $\vec{i}$ and $\underline{\mathcal{AB}} \subseteq \mathcal{AB}$ such that

$$\sum_{(A_j,B_j)\in\mathcal{AB}} R_j|\overrightarrow{A_jB_j}| \leq K_d \sum_{(A_j,B_j)\in\underline{\mathcal{AB}}} R_j\overrightarrow{A_jB_j}\cdot\vec{i} \qquad (14)$$

$$C_T^{\mathrm{nc}} = \max_{\mathcal{AB}}\Big(\sum_{(A_j,B_j)\in\mathcal{AB}} R_j|\overrightarrow{A_jB_j}|\Big) \leq K_d \cdot C_T^{\mathrm{f}}$$

Conclusion

- This paper provides the fundamental limitations of the benefit of network coding.