On Achieving Optimal End-to-End Throughput in Data Networks: Theoretical and Empirical Studies

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Introduction

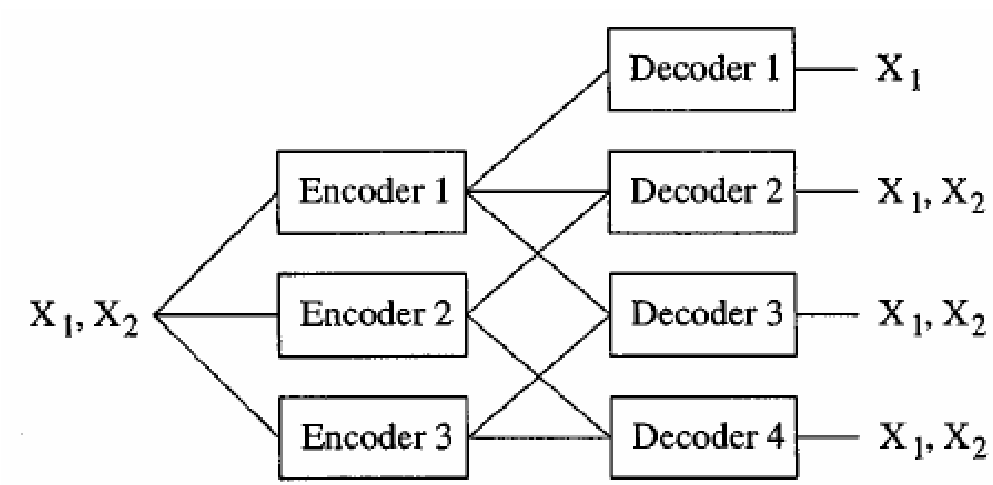
- The throughput of information transmission within a data network is constrained by the network topology and link capacities.
- Improve transmission throughput
  - Traditional techniques: routing strategy
  - New dimension: network coding
- Optimal transmission strategy to achieve max throughput includes both a routing scheme and a corresponding coding scheme.

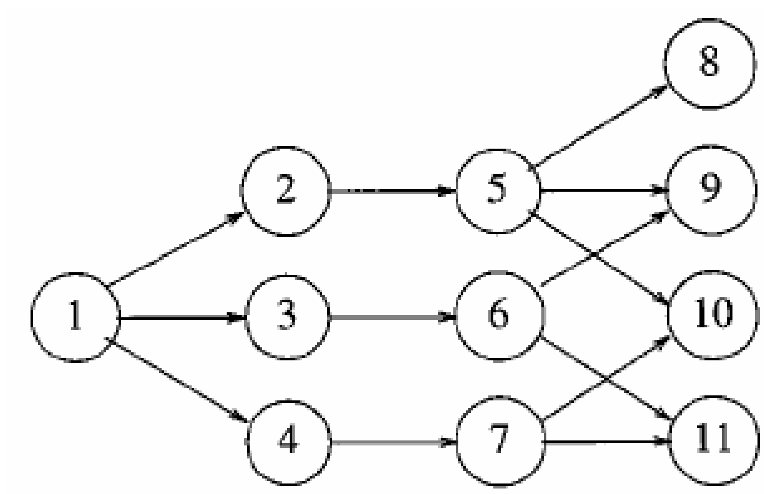
What is Network Coding (1/2)

- In existing computer networks, each node functions as a switch that either
  - Relays information from an input link to an output link, or
  - Replicates information from an input link and sends to a certain set of output links
- A node can function as an encoder in the sense that it receives information from all input links, encodes, and sends information to all output links

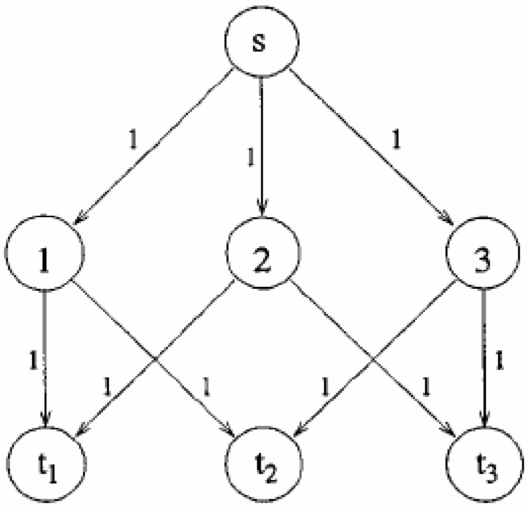
What is Network Coding (2/2)

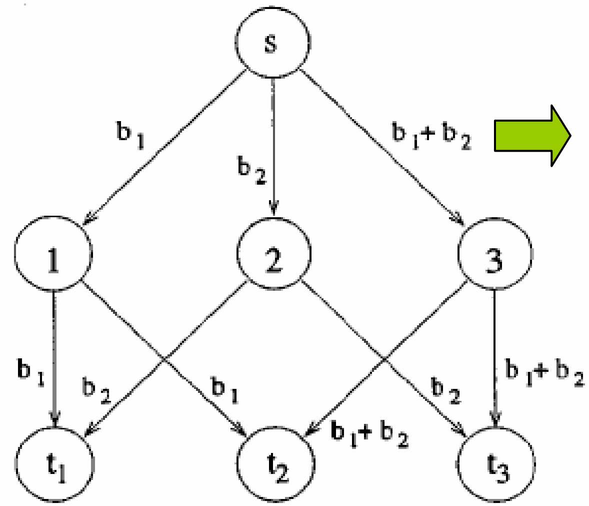
- Directed graph $G=(V,E)$, let $\boldsymbol{R}=[R_{ij}, (i,j)\in E]$
  - A vector $\boldsymbol{R}$ is admissible if and only if there exists a coding scheme satisfying the set of multicast requirements such that the coding rate from node i to node j is less than or equal to R_{ij} for all $(i, j)\in E$





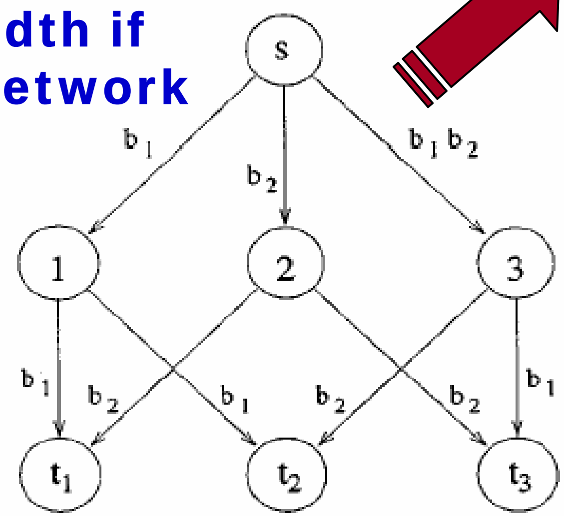
Advantage of Network Coding (1/2)



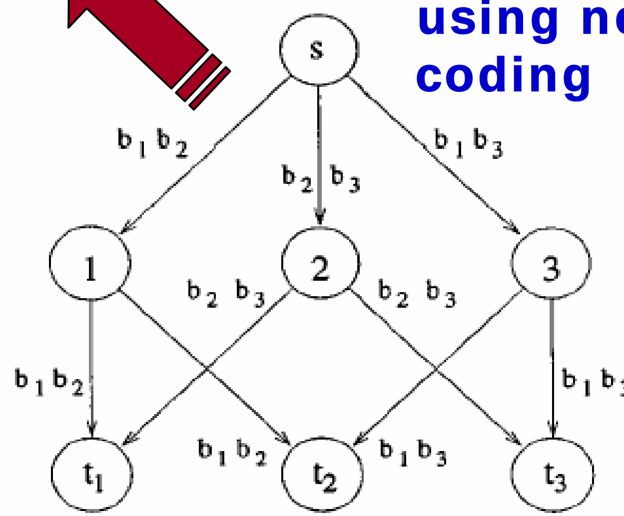


Simple coding: b_1 or b_2

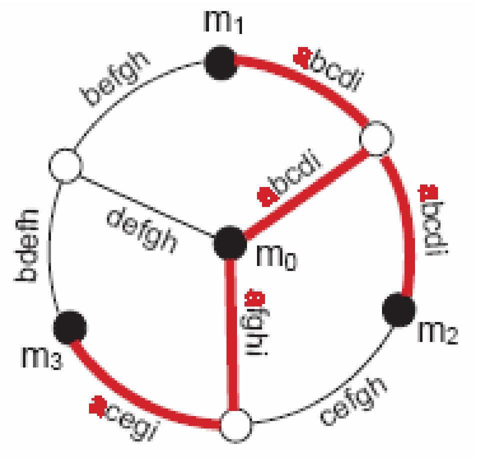
Save 10% in Bandwidth if using network coding



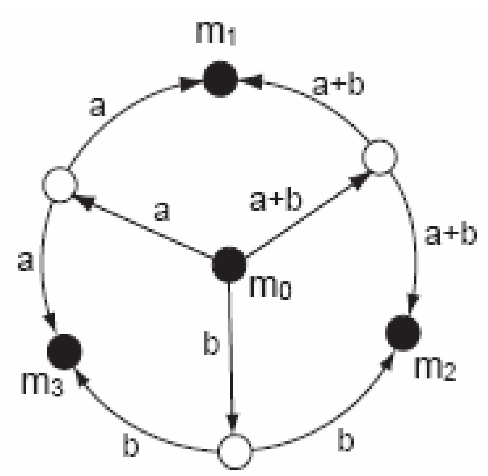
Increase the throughput by one-third if using network coding



Advantage of Network Coding (2/2)



(a) steiner tree packing and multicast without coding.



(b) multicast with network coding.

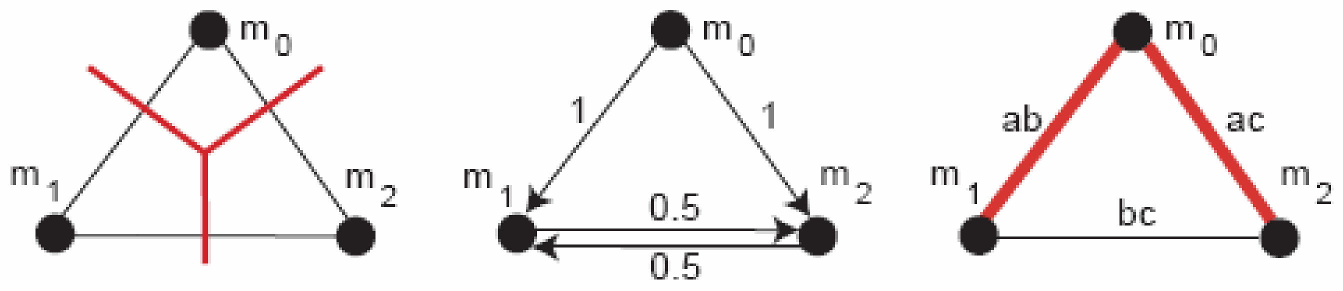
- State-of-the-art approaches to compute the optimal throughput of multicast sessions
  - Steiner tree packing
  - Steiner strength
- Both Steiner tree packing and Steiner strength have been shown to be NP-complete
- The achievable optimal throughput
  - 1.8 without coding (Steiner tree packing)
  - 2.0 with simple network coding

Steiner Tree Packing and Steiner Strength

- A Steiner tree is a sub-tree of the network that connects every multicast node. The network is decomposed into weighted steiner trees such that the total of tree weights is maximized, referred to as the **steiner tree packing number**
- In an undirected network N, let P be the set of network partitions where there exists at least one source or receiver node in each component of the partition, the **steiner strength** of N is defined as $\min_{p \in P}|E_c|/(|p|-1)$, where $|E_c|$ is the total inter-component capacity on the set of link E_c being cut, and $|p|$ is the number of components in the partition p

Directed vs. Undirected Network

- In directed networks, if a rate x can be achieved for each receiver in the group independently, it can also be achieved for the entire multicast session.
- For an undirected data network N with a single multicast session, $\pi(N) \leq \chi(N) \leq \eta(N)$, where
  - $\pi(N)$ is the steiner tree packing number
  - $\chi(N)$ is the achievable optimal throughput
  - $\eta(N)$ is the steiner strength



(a) A *partition*. (b) Optimal throughput with network coding is 1.5. (c) Optimal throughput without network coding is also 1.5.

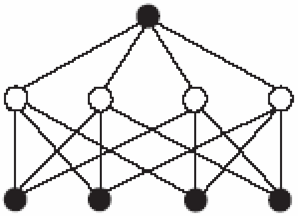
cFlow Linear Program

- *cFlow* (conceptual flow): network flows that co-exist in the network without contending for link capacities
- *cFlow* LP: compute optimal throughput with network coding
  - m_0 is the source and $m_1 \ldots m_k$ are the multicast receivers
  - $f^1 \ldots f^k$ are conceptual flows from sender m_0 to each of the receivers; $f^i(a)$ is the flow rate for each directed link a
  - Maximize f^* under constraints of
    - Orientation constraints
    - Independent network flow constraints for each conceptual flow
    - Equal rate constraints $f^* = f_{in}^i(m_i) \quad \forall i \in [1 \ldots k]$
  - $f^* = \max_{o \in O}[\min_{m_i \in M - \{m_0\}} (\text{maximum } m_0 \rightarrow m_i \text{ flow rate})]$
  - Complexity: O(|M| · |E|)

cFlow LP vs. Steiner Tree Packing

Network	$\|V\|$	$\|M\|$	$\|E\|$	$\chi(N)$	$\pi(N)$	$\frac{\chi(N)}{\pi(N)}$	# of trees
Fig. 1	7	3	9	2	1.875	1.067	17
$C(3,2)$	7	4	9	2	1.8	1.111	26
$C(4,3)$	9	5	16	3	2.667	1.125	1,113
$C(4,2)$	11	7	16	2	1.778	1.125	1,128
$C(5,4)$	11	6	25	4	3.571	1.12	75,524
$C(5,2)$	16	11	25	2	1.786	1.12	119,104
$C(5,3)$	16	11	35	3	–	–	49,956,624

- *cFlow* LP is much more scalable and efficient than Steiner tree packing



- Optimal throughput with coding is always lower bounded by that without coding, however, $\chi(N)/\pi(N)$ no higher than 1.125
- Coded transmission may lead to more integral flow rates and throughput than uncoded transmission

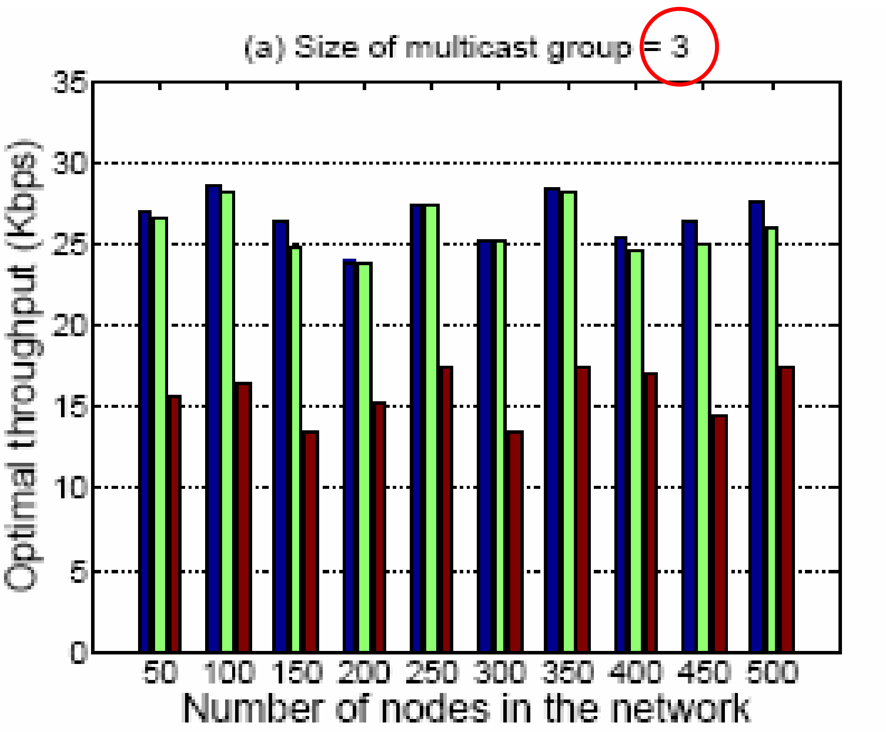
mFlow LP and *oFlow* LP

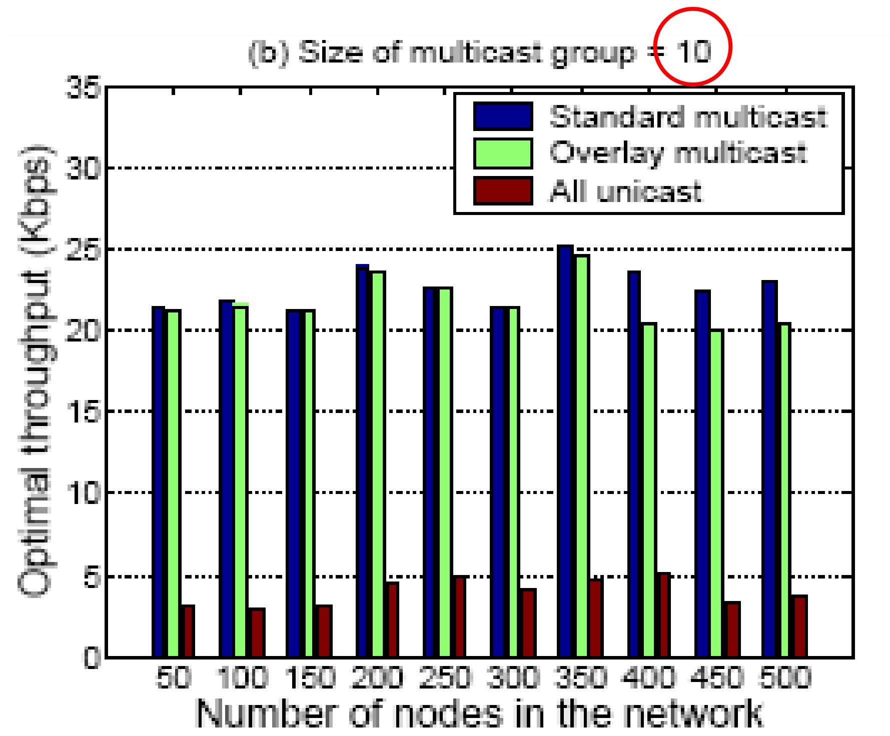
- *mFlow* LP
  - Compute optimal throughput for multiple sessions
  - Replace standard network flow constraints in *cFlow* LP with a set of multicommodity *cFlow* constraints
  - Complexity: $O(s \cdot |M| \cdot |E|)$
- *oFlow* LP
  - Compute optimal throughput for the overlay networks
  - Standard multicommodity flow constraints are specified for the underlay flows between end hosts and only via routers
  - Map the underlay flow rate to the overlay link capacity and then apply original *cFlow* constraints in the overlay level
  - Complexity: $O(|H|^2 \cdot |E|)$, H: set of overlay nodes

How advantageous is network coding with respect to improving optimal throughput

- Coding advantage: the ratio of achievable optimal throughput with coding over that without coding
- Previous work shows that in directed acyclic networks with integral routing requirements, there exist multicast networks where the coding advantage grows proportionally as log(|V|)
- For multicast transmission in undirected networks, the coding advantage is bounded by a constant factor of 2
- The fundamental benefit of network coding is not higher optimal throughput, but to facilitate significantly more efficient computation and implementation of strategies

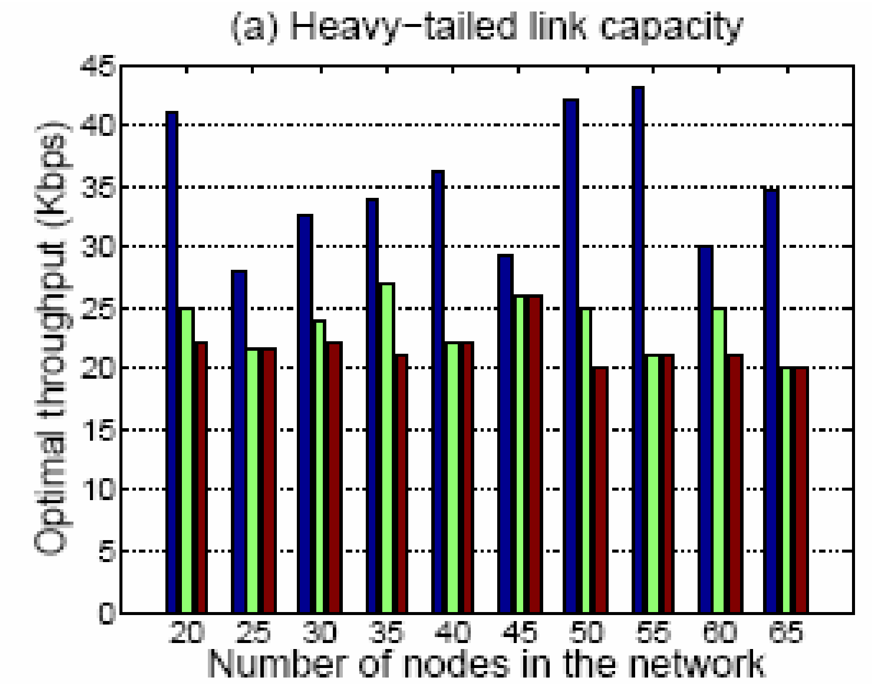
How advantageous is standard multicast compared to unicast and overlay multicast?

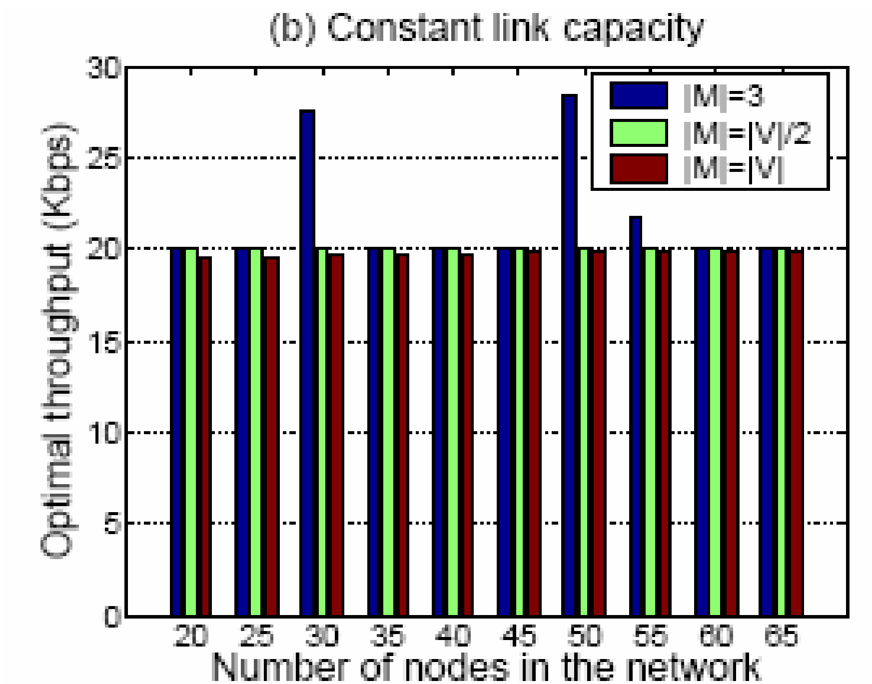




- The optimal throughput achieved by overlay multicast is almost identical to that achieved by standard multicast

How sensitive is optimal throughput to node joins?

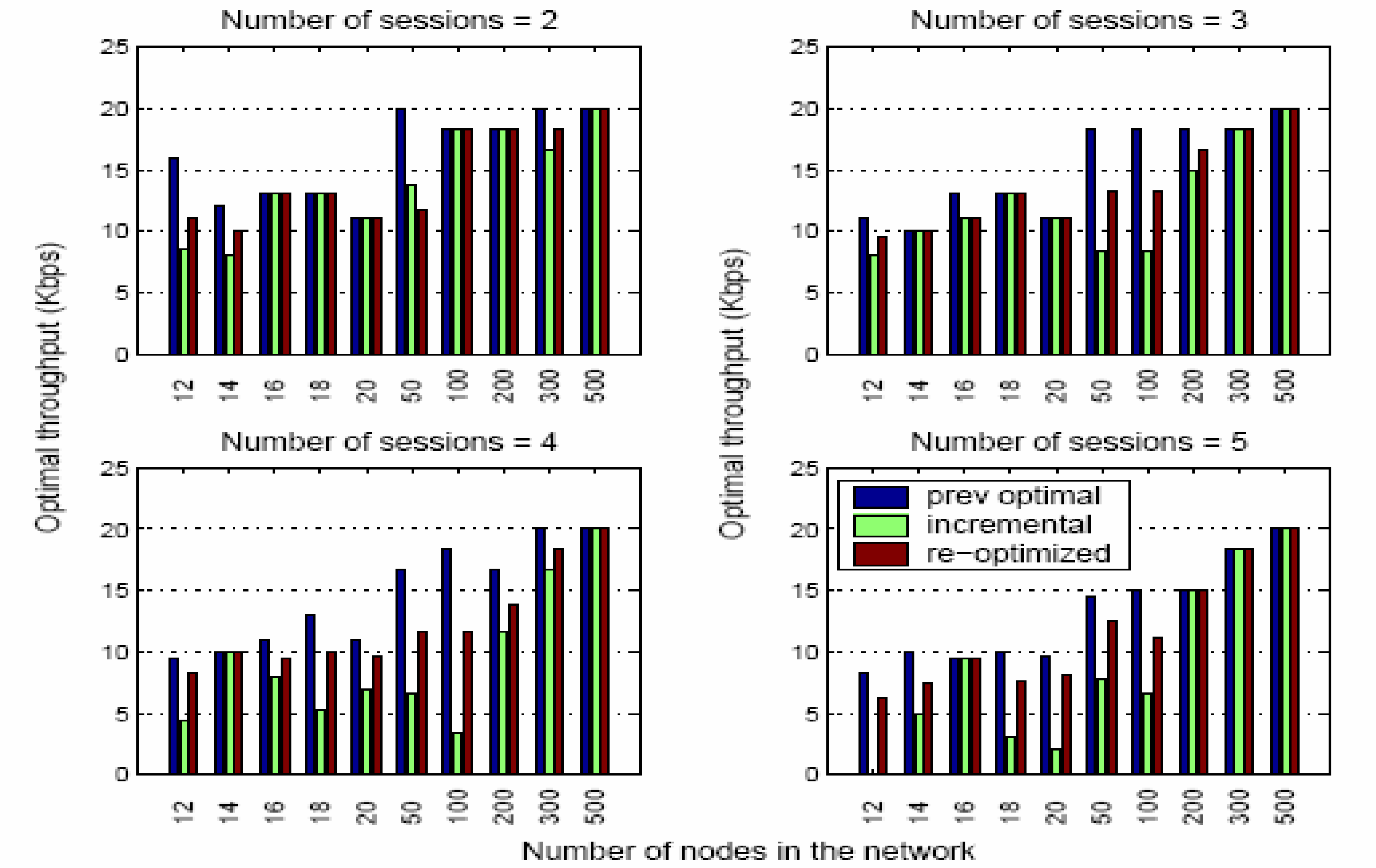




- Effect on achievable throughput is rather significant for multicast group from 3 to |V|/2

- Power-law network topologies
- Optimal multicast throughput remains roughly constant

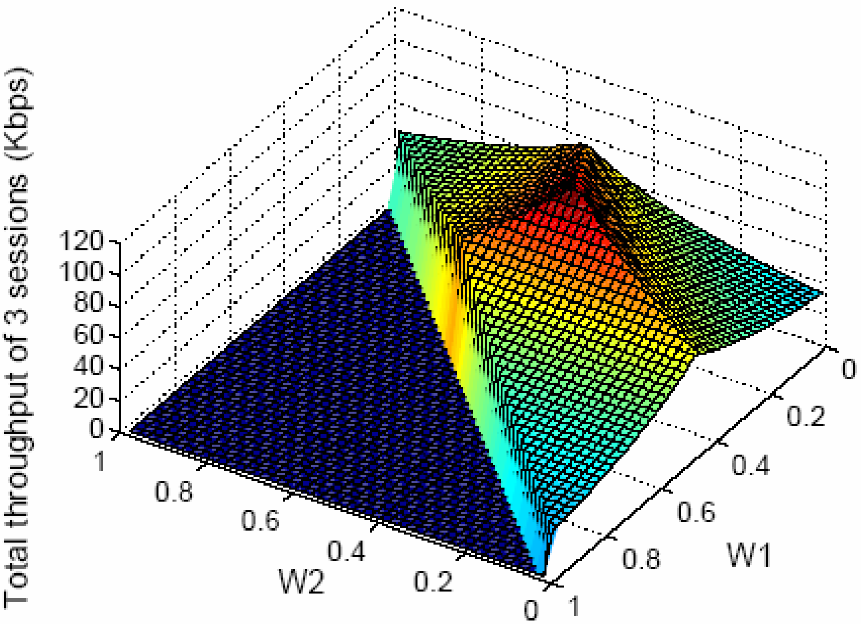
How sensitive is optimal throughput to the addition of new sessions?



- Addition of extra session does not dramatically affect achievable optimal throughput, especially when network size is large

How sensitive is optimal throughput to fairness constraints?

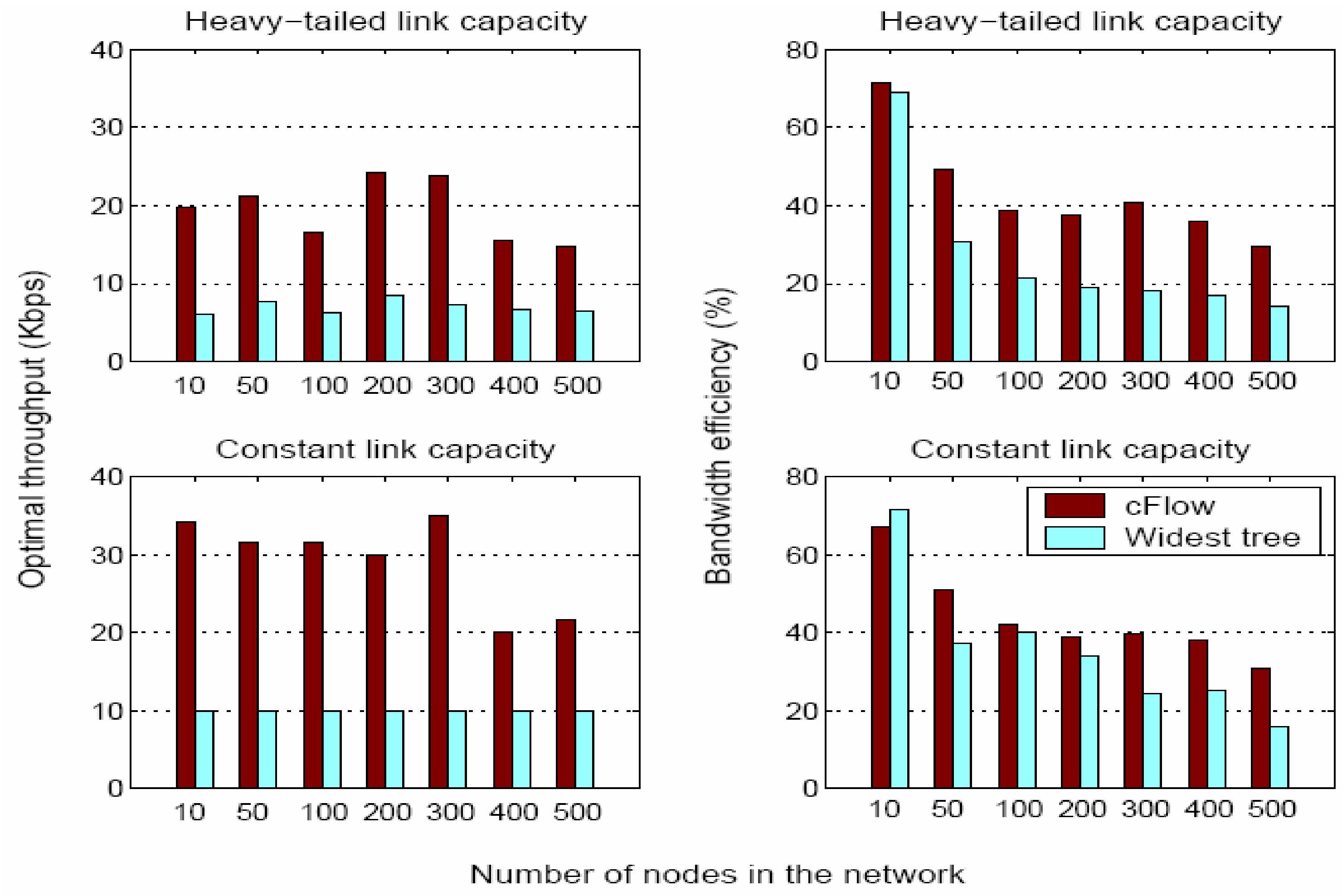
- 3 1-to-2 multicast sessions, 4 types of fairness constraints are investigated
  - No fairness requirement
  - Absolute fairness
  - Weighted proportional fairness
  - Max-min fairness
- Global optimal throughput: $(w_1,w_2,w_3)=(0.287,0.407,0.306)$ which is identical to the throughput with max-min fairness



network size	10	20	50	100	150	250	350
max-min (Kbps)	120.0	136.7	173.3	160.0	146.7	146.7	183.3
optimal (Kbps)	126.1	140.0	173.3	160.0	146.7	146.7	183.3

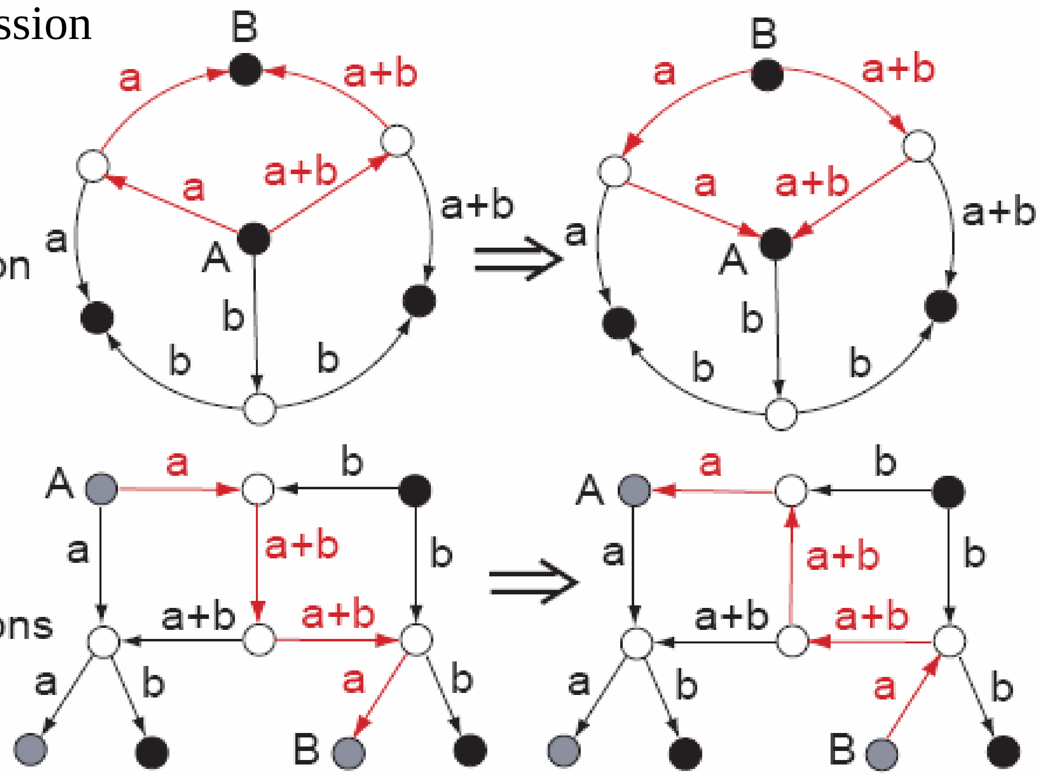
Does optimal throughput lead to low bandwidth efficiency?

- Bandwidth efficiency of optimal throughput multicast is also better than that of the widest steiner tree multicast



Is the optimal throughput dependent on the selection of data sources in a session?

- For an undirected data network with one or more communication sessions, the achievable optimal throughput is completely determined by
  - Network topology
  - Link capacities
  - Set of nodes in each session
  - Inter-session fairness
  - Irrelevant to the selection of data source



(a) Source independence: the case of a single session

(b) Source independence: the case of multiple sessions

Conclusions

- This paper focuses on solving the problem of computing and achieving optimal throughput in data networks, in the general case of undirected links.
- The most significant benefit of network coding is not to achieve higher optimal throughput, but to make it feasible to achieve such optimality in polynomial time.
- Efficient algorithms are designed for multiple communication sessions of a variety of types, and for the more realistic model of overlay network.